OCR Maths S1

Topic Questions from Papers

Arrangements and Combinations

Answers

1 (i)	$\frac{2\times 7!}{8!}$ $= ¼$	M1 M1 A1 3	7! and 8! used or ${}^{7}P_{7}$ and ${}^{8}P_{8}$ Correct formula, with "2 ×" Answer, $^{1}/_{4}$ or 0.25 only
(ii)	$^{1}/_{4}$ or $4!\times 4!$ or $3!\times 3!$ or $^{3!}/_{4!}$ $\left(\frac{1}{4}\right)^2$ or $\frac{3!\times 3!}{4!\times 4!}$ $= {}^{1}/_{16}$	M1 M1 A1 3	 Correct expression or 0.0625
(iii) Attempt subdivide, allow one error. Correct subdivision into 3 or 13 cases Correct expression $= \frac{13}{16}$		M1 M1 M1 A1 4	By description or listing or implied by probs, eg 1 – (ii) – P(sep by 1) All 3 or all 13 cases clearly present or 0.8125 or a.r.t. 0.813 only
Eg correct: $1-3\times\frac{1}{16}$; $1-(ii)-2\times\frac{3!\times 3!}{4!\times 4!}$ $\frac{3!\times 3!\times 13}{(4!\times 4!)}$; $(^{3}/_{4})^2 + 2\times {}^{1}/_{4}\times {}^{2}/_{4}$			Eg incorrect: $1-\frac{3!\times 3!\times 3}{8!}$: M1M1M0A0 $1-{}^{1}/_{16}-\frac{3!\times 3!}{4!\times 4!}$: M1M0M0A0

(Q8, Jan 2005)

2 (i) ${}^{18}C_7$ or ${}^{18!}/_{(11!\ x7!)}$	M1		
= 31824	A1	2	cao
(ii) ${}^5C_2 \times {}^6C_2 \times {}^7C_3$ or 5250	M2		M1: 1 correct ${}^nC_r$ or mult any three ${}^nC_r$s
÷ 31824	M1		Divide by their (i). Indep
= 875/5304 or 5250/31824 oe			If cancelled, must be clear have ÷ 31824
or 0.165 (3 sfs)	A1	4	
			5 x 4 x 6 x 5 x 7 x 6 x 5 x 7! / 18x17x 16 x 15 x 14 x 13 x 12 x $2!^2$x3!
			Correct 7 fractions mult: M1
			x7!: M1}
			÷ ($2!^2$x3!): M1}both dep any 7 fracts mult
(iii) 5 from W & 2 from (G + H)	M1		Seen or implied, eg by combs or list
${}^7C_5 \times {}^{11}C_2$ or 1155	M1		
÷ 31824	M1		Divide by their (i). Indep
= 385/10608 or 1155/31824 oe		4	
or 0.0363 (3 sfs)	A1		
			7 x 6 x 5 x 4 x 3 x 11 x 10 x 7! / 18x17x16 x 15 x14 x 13 x 12 x 5! x 2!
			Correct 7 fractions mult: M1
			x 7! : M1}
			÷ (5! x 2!): M1} both dep any 7 fracts mult
(iv) (2, 2, 3) or (2, 3, 2) or (3, 2, 2)	M1		Any one. Seen or implied eg by combs
${}^5C_2\times{}^6C_2\times{}^7C_3+{}^5C_2\times{}^6C_3\times{}^7C_2$			M1: one correct product.
$+{}^5C_3\times{}^6C_2\times{}^7C_2$	M2		NOT ${}^5C_2\times{}^6C_2\times{}^7C_2$
(÷ 31824)			(No mk for ÷ 31824)
= 175/442 or 12600/31824 oe			
or 0.396 (3 sfs)	A1	4	
			~~Equiv method; ((ii) + etc) can imply M mks~~
			5 x 4 x 6 x 5 x 7 x 6 x 7! / 18x17x16 x 15 x14 x 13 x $2!^2$x3!
			Correct 6 fractions mult: M1
			x 7! : M1}
			÷ ($2!^2$ x 3!) : M1}both dep any 6 fracts mult
			Complement method:
			Triple with total 7, incl at least one 0 or 1
			or (0, 7) or (1, 6) seen or implied: M1
			One correct prod seen, eg ${}^5C_0x{}^6C_2x{}^7C_5$ M1
			Full correct method, incl "1 – " M1
	14		

(Q7, June 2005)

3 (i)	$\frac{7!}{3! \times 2(!)}$ = 420	M1M1 A1 **3**	M1: 7!/(a factorial); or … ÷ (3! x 2(!)) M1: all correct
(ii)	$\frac{5!}{2(!)}$ = 60	M1 A1 **2**	M1: 5! seen (not part of a C) or 5 x 4! or 120 seen or … ÷ 2(!) alone
(iii)	$1 - {}^4/_7 \times {}^3/_6$ or $1 - {}^4C_2 / {}^7C_2$ or $1 - {}^4P_2 / {}^7P_2$ or ${}^3/_7 \times {}^2/_6 + {}^3/_7 \times {}^4/_6 + {}^4/_7 \times {}^3/_6$ oe or ${}^3C_2 / {}^7C_2 + {}^3C_1 \times {}^4C_1 / {}^7C_2$ $= {}^5/_7$ or 0.714 (3 sfs)	M1M1 A1 **3**	M1:1– prod or 1-.../${}^7C_2$ or 1–${}^4C_2$ /.. (or Ps) or add 3 prods or add 2 correct prods or ${}^3C_2 / {}^7C_2$ or ${}^3C_1 \times {}^4C_1 / {}^7C_2$ or add $\geq$ 5 out of 7 correct prods M1: all correct
Total		**8**	

(Q3, June 2006)

4 (i)	120	B1 1	not just 5!
iia	3 x 4! or 72 (÷ 5!) ${}^3/_5$ oe	M1 A1 2	oe, eg ${}^{72}/_{120}$
b	Starts 1 or 21 (both) ${}^1/_5 + {}^1/_5 \times {}^1/_4$ $= {}^1/_4$ oe	M1 M1 A1 3	12,13,14,15, (≥2 of these incl 21, or allow 1 extra) can be implied by wking or 5x 3! or 4! + 3! (÷5!) complement: full equiv steps for Ms
Total		**6**	

(Q3, Jan 2007)

5 (i)	${}^{15}C_7$ or ${}^{15!}/_{7!8!}$ 6435	M1 A1 2	
ii	${}^6C_3 \times {}^9C_4$ or ${}^{6!}/_{3!3!} \times {}^{9!}/_{4!5!}$ 2520	M1 A1 2	Alone except allow ÷ ${}^{15}C_7$ Or ${}^6P_3 \times {}^9P_4$ or ${}^{6!}/_{3!} \times {}^{9!}/_{5!}$ Allow ÷ ${}^{15}P_7$ NB not ${}^{6!}/_{3!} \times {}^{9!}/_{4!}$ 362880
Total		**4**	

(Q3, June 2007)

6 (ia)	5! or $^5P_5$ = 120	M1 A1 2	
b	4! or $^4P_4$ seen 4! × 2 48	M1 M1dep A1 3	or 2 × 3! or 2! × 3! or 2! × $^3P_3$ 2 × 3! × 4
ii	$^1/_5C_2$ or $^1/_5$ × ¼ × 2 or 0.4 × 0.25 or $^2/_{5P2}$ = $^1/_{10}$	M1 A1 2	Allow M1 for $^5C_2$ or $^1/_5$ x ¼ or $^1/_{20}$ or $^1/_5 \times ^1/_5 \times 2$ or $^2/_{25}$ oe
Total		**7**	

(Q1, Jan 2008)

7 (ia)	8! = 40320	M1 A1 2	Allow $^4P_4$ & $^3P_3$ instead of 3! & 4! thro'out Q6
(b)	$^4/_8 \times ^4/_7 \times ^3/_6 \times ^3/_5 \times ^2/_4 \times ^2/_3 \times ^1/_2$ × 2 = $^1/_{35}$ or 0.0286 (3 sfs)	M1 M1dep A1 3	4! × 4! ÷ 8! × 2 allow 1 – above for M1 only oe, eg $^{1152}/_{40320}$ | 4! × 4! + 4! × 4! ÷ 8!
(ii)(a)	4! × 4! = 576	M1 A1 2	allow 4! × 4! × 2: M1
(b)	$^1/_{16}$ or 0.0625	B1 1	
(c)	Separated by 5 or 6 qus stated or illus $^1/_4 \times ^1/_4 \times 3$ or $^1/_{16} \times 3$ ($^1/_4 \times ^1/_4$ or $^1/_{16}$ alone or ×(2 or 6): M1) $^3/_{16}$ or 0.1875 or 0.188	M1 M2 A1 4	allow 5 only or 6 only or (4, 5 or 6) can be impl by next M2 or M1 3! × 3! × 3 (3!× 3! alone or ×(2 or 6); or (3! + 3!) × 3: M1) (÷ 576) correct ans, but clearly B, J sep by 4: M0M2A0 1- P(sep by 0, 1, 2, 3, (4)) M1 $1-(^1/_4+^1/_4+^1/_4\times^3/_4+^1/_4\times^1/_2)$ or $1-(^1/_4\times^1/_4+^1/_2\times^1/_4+^3/_4\times^1/_4+1\times^1/_4+^3/_4\times^1/_4)$ M2 (one omit: M1)
Total		**12**	

(Q6, Jan 2009)

8 (i)	$^8C_3$ = 56		M1 A1 2		
ii	$^7C_2$ or or $^7P_2$ / $^8P_3$ ÷($^8C_3$ or "56") only = $^3/_8$	$^1/_8$ not from incorrect × 3 only or $^1/_8+^7/_8\times^1/_7+^7/_8\times^6/_7\times^1/_6$	M1 M1 A1 3	$^8C_1+^7C_1+^6C_1$ or 21 or 8×7×6 or $^{\cdot\cdot}/_8\times^{\cdot\cdot}/_7\times^{\cdot\cdot}/_6$ indep, dep ans < 1	$^7/_8\times^6/_7\times^5/_6$ 1 – prod 3 probs
iii	$^8P_3$ or 8×7×6 or $^8C_1\times^7C_1\times^6C_1$ or 336 1 ÷ $^8P_3$ only = $^1/_{336}$ or 0.00298 (3 sf)		M1 M1 A1 3	$^1/_8\times^1/_7\times^1/_6$ only M2 If × or ÷: M1 $(^1/_8)^3$ M1	
Total			**8**		

(Q7, June 2009)

9 (i)	$^{5!}/_2$ = 60	M1 A1 2	Allow 5P3
(ii)	4! = 24	M1 A1 2	Allow 2×4!
(iii)	$^2/_5\times^3/_4$ or3/5×2/4 × 2 = $^3/_5$ oe	M1 M1 A1 3	allow M1 for $^2/_5\times^3/_5\times 2$ or $^{12}/_{25}$ or (6×3!)÷**(i)** M2 or 3!÷**(i),**6÷**(i)**,(6+6)÷**(i)**,6k÷**(i)** or 6×6 or 36 or 1-correct answer M1 (k,integer ≤5)
Total		**[7]**	

(Q8, Jan 2010)

10 (i)	$^4C_2\times^6C_3\times^5C_4$ or 6×20×5 = 600	M1M1 A1 3	M1 for any 2 correct combs seen, even if added
ii	$\frac{2}{4}$ or $\frac{^3C_1}{^4C_2}$ or $\frac{^3C_1\times^6C_3\times^5C_4}{^4C_2\times^6C_3\times^5C_4}$ or $\frac{^3C_1\times^6C_3\times^5C_4}{'600'}$ = $\frac{1}{2}$ oe	M1 A1 2	or $\frac{1}{4}\times 1+\frac{3}{4}\times\frac{1}{3}$ or $\frac{1}{4}\times 2$ or $\frac{1}{4}+\frac{1}{4}$
iii	$^3C_1\times^6C_3$ (×$^4C_4$) + $^3C_2\times^6C_3\times^5C_4$ 360	M1M1 A1 3	M1 either product seen, even if × or ÷ by something
Total		**8**	

(Q7, June 2010)

11 (i)	7! ÷ 3! 7! ÷ 2! ÷ 2! ÷ 3! = 420	M1 M1dep A1 3	But NOT $^7P_4$ or 7!/(7-4)! if seen	$\frac{7!}{3!+2!}$: M1M0 $\frac{7!}{3!\times n!}$ any n: M1M0
iia	$^5C_3$ or $^{10}C_4$ seen $^5C_3 \times {}^{10}C_4$ = 2100	M1 M1 A1 3	or 10 or 210	$\frac{^5C_3\times {}^{10}C_4}{\text{anything}}$ M1M1A0 $^5P_3 \times {}^{10}P_4$ or 60 × 5040 or 302400: SC B1
b	$^4C_2 \times {}^9C_4$ or $^4C_3 \times {}^9C_3$ or 756 or 336 $^4C_2 \times {}^9C_4 + {}^4C_3 \times {}^9C_3$ or 1092 ÷ 2100 or ÷ (iia) dep ≥ one M1 scored = $\frac{13}{25}$ or 0.52	M1 M1 M1dep A1 4	$\frac{3}{5}$ or $\frac{4}{10}$ oe $\frac{3}{5}\times(1-\frac{4}{10})$ or $(1-\frac{3}{5})\times\frac{4}{10}$ $\frac{3}{5}\times(1-\frac{4}{10}) + (1-\frac{3}{5})\times\frac{4}{10}$ = $\frac{13}{25}$	Not from incorrect wking SC $\frac{1}{5}\times\frac{9}{10}$ or $\frac{4}{5}\times\frac{1}{10}$ M1 $\frac{1}{5}\times\frac{9}{10} + \frac{4}{5}\times\frac{1}{10}$ M1 (= $\frac{13}{50}$ A0)
	"2100" – ($^4C_3 \times {}^9C_4$ or $^4C_2 \times {}^9C_3$) or "2100" – (504 or 504) M1 "2100" – ($^4C_3 \times {}^9C_4 + {}^4C_2 \times {}^9C_3$) M1 ÷ "2100" or (iia) dep ≥ M1 M1		$\frac{3}{5}$ or $\frac{4}{10}$ oe M1 $\frac{3}{5}+\frac{4}{10}-\frac{3}{5}\times\frac{4}{10}$ M1 $\frac{3}{5}+\frac{4}{10}-\frac{3}{5}\times\frac{4}{10}-\frac{3}{5}\times\frac{4}{10}$ M1 = $\frac{13}{25}$ A1	Not from incorrect wking ie P(WA or GA or both) Must be correct figures ie P(WA or GA but not both) Must be correct figures SC : $^4P_2 \times {}^9P_4 + {}^4P_3 \times {}^9P_3$: M1 ÷ (iia) M1dep Careful: 336 or 756 can be obtained by incorrect methods.
Total		**10**		

(Q6, Jan 2011)

12 (ia)	5040	B1 1		
b	6! or 5!×6 or 720 ÷ 7! or ÷ "5040" or 1440 or (5! or 6!) × 2 = $^2/_7$ oe or 0.286 (3 sf)	M1 M1 A1 3	Any ÷ 7! or "5040" but NOT any × 2	$^1/_7\times{}^1/_6$ M1* × 6 or × 2 M1 dep*	NOT 6! in denom eg $^{6!}/_{5040}$ or $^1/_7$ or 0.143 or $^1/_{21}$ (3 sfs): M1M1A0
iia	3! × 4! alone or 144 (÷ 7! or "5040") = $^1/_{35}$ oe or 0.0286 (3sf)	M1 A1 2	$^4/_7\times{}^3/_6\times{}^3/_5\times{}^2/_4\times{}^2/_3\times{}^1/_2$ oe or $\frac{1}{7C3\text{or}7C4}$	Not 3! × 4! ×… (eg not 3! × 4! × 5) not $\frac{1}{3!\times4!}$, not $\frac{1}{144}$ NB no mark for ÷ 7! or "5040" in this part
b	5 seen or 5! seen 3! × 4! × 5 or 5! × 3! or 720 or 5 × 144 (÷7! or "5040") = $^1/_7$ oe or 0.143 (3 sf)	M1 M1 A1 3	or 5 × $^3/_7\times{}^2/_6\times{}^1/_5$ ($\times{}^4/_4\times{}^3/_3\times{}^2/_2$) oe: M2 or 5 × $\frac{1}{7C3\text{or}7C4}$: M2 or 5 × "(iia)": M2	or GGGBBBB, BGGGBBB, BBGGGBB, BBBGGGB, BBBBGGG NB no mark for ÷ 7! or "5040" in this part
Total		**9**		

(Q6, June 2011)

13	(i)	(a)	$^9P_4$ or $^{9!}/_{5!}$ or $^9C_4 \times 4!$ = 3024	M1 A1 [2]	alone	oe eg $^9C_1 \times {}^8C_1 \times {}^7C_1 \times {}^6C_1$ or $9\times8\times7\times6$
	(i)	(b)	$^8P_3$ or $8 \times 7 \times 6$ oe or $^8C_3 \times 3!$ $\times 5$ (or $^5C_1$) = 1680	M1 M1 A1 [3]	Allow × …. or ÷ ….. Correct × 5 or $^8C_3 \times 5$ (or $^5C_1$) Not ISW, eg $^{1680}/_{3024}$: M1M1A0	or ($^9P_4$ or "3024") × $^5/_9$ M2
					SC: consistent use of with replacement in (i) (or if only (a) or (b) attempted) (ia) M0A0 (ib) 999 × 5 or 4995 M1 M0A0	
	(ii)	(a)	$^5C_3 \times {}^4C_1$ or $^5C_4$ oe $^5C_3 \times {}^4C_1 + {}^5C_4$ oe correct method so far (= 45) $\div {}^9C_4$ Allow anything $\div {}^9C_4$ = $^5/_{14}$ or 0.357 (3 sfs) oe, eg $^{35}/_{98}$ or $^{45}/_{126}$	M1 M1 M1 A1 [4]	$^5C_3 \times {}^4C_1 \times 4!$ (or $^5P_3 \times 4 \times 4$) or 5! (or $^5P_4$) 960 + 120 oe correct method so far $\div {}^9P_4$ [must involve any P or any !] $\div {}^9P_4$ Marks must come from one method, not mixture of two methods	$^5/_9 \times {}^4/_8 \times {}^3/_7 \times {}^4/_6$ Allow × or +… × 4 correct method so far $^5/_9 \times {}^4/_8 \times {}^3/_7 \times {}^2/_6$ Allow × or +… or: $^5/_9 \times {}^4/_8 \times {}^3/_7 \times {}^4/_6$ or $^5/_9 \times {}^4/_8 \times {}^3/_7$ M1 $^5/_9 \times {}^4/_8 \times {}^3/_7 \times {}^4/_6 \times 3 + {}^5/_9 \times {}^4/_8 \times {}^3/_7$ M1 NB $^5/_9 \times {}^4/_8 \times {}^3/_7 \times 3 = {}^5/_{14}$ M0M0M0A0
	(ii)	(b)	9, 8, 7, 4 or 9, 8, 6, 5 No mark yet 2 $\div {}^9C_4$ oe Must be (1 or 2 or 4) $\div {}^9C_4$ = $^1/_{63}$ oe or 0.0159 (3 sfs)	 M1 M1 A1 [3]	$^1/_9 \times {}^1/_8 \times {}^1/_7 \times {}^1/_6$; $^4/_9 \times {}^3/_8 \times {}^2/_7 \times {}^1/_6$ Allow × or +… × 4! × 2 ; × 2 fully correct method NB Marks from one method only, not mixed methods	4! + 4! or 2 × 4! oe $\div {}^9P_4$ or ÷ (i)(a) oe Must be (96 or 48 or 24) $\div {}^9P_4$ $^2/_9 \times {}^2/_8 \times {}^1/_7 \times {}^1/_6$ allow × or + … M1 × 4!/4 × 2 fully correct method M1
					SC: consistent use of with replacement in (ii), (or if only (a) or (b) attempted) (iia) $(^5/_9)^4$ M1 + $^4C_3(^5/_9)^3(^4/_9)$ (= 0.400) M1 M0A0 (iib) $(^1/_9)^4$ (=0.000152) M1 attempt find no of gps M1A0	$1-((^4/_9)^4+4(^4/_9)^3(^5/_9)+{}^4C_2(^4/_9)^2(^5/_9)^2)$ M2 One term missing or extra or wrong M1

(Q9, Jan 2012)

14	(i)	(a)	$^7P_5$ or $\frac{7!}{2!}$ or $7\times6\times5\times4\times3$ or $^7C_5 \times 5!$ alone = 2520	M1 A1 **[2]**	$^7P_2$ or $\frac{7!}{2!}$ M0A0	$^7C_5 = 21$ or 5! = 120 M0A0 but see (i)(b)
	(i)	(b)	$^6P_4$ or $\frac{6!}{2!}$ or $6\times5\times4\times3$ or $^6C_4 \times 4!$ or 360 × 2 (see middle column) = 720	M1 M1 A1 **[3]**	alone or ×2 only $^6P_4 \times 2$ or 6! alone M2 $^6C_4 \times 2$ or 6! × 2 alone M0M1 only any other × 2 M0M0 or '2520' × $\frac{2}{7}$ M2A0 (eg (ia)21 (ib) $21\times\frac{2}{7}=6$ M2A0 but if ans is 6, must see wking) cao	or '2520' – 5 × $^6P_4$ M2 SC ONLY on ft from (i)(a): if (i)(a) 5! = 120, then (i)(b)4!×2=48 alone M1M0A0 Other SC $^5P_3 \times 2$ M2 (from a vowel at <u>each</u> end, ie treat as MR) NOT isw eg $\frac{720}{'2520'} = \frac{2}{7}$ M1M1A0
	(ii)	(a)	21	B1 **[1]**		
	(ii)	(b)	$^5C_3$ or $\frac{5!}{3!2!}$ or $^5C_5$ seen or 10 seen in num $\frac{{}^5C_3}{{}^5C_3+{}^5C_5}$ oe $\frac{10}{11}$ or 0.909 (3 sf)	M1 M1 A1 **[3]**	$\frac{5}{7}\times\frac{4}{6}$ oe seen $\frac{5}{7}\times\frac{4}{6}\div(\frac{5}{7}\times\frac{4}{6}+\frac{2}{7}\times\frac{1}{6})$	Allow $^5C_2$ seen BOD

(Q7, June 2012)

15	**(i)**	**(b)**	3×3×3 = 27	M1 A1 **[2]**	3! + 7×3 3 + 3×6 + 6 3! × 4 + 3 Complete correct method. Allow methods equiv to these. Only allow other methods if they appear correct	(Explanation for 3! × 4 + 3: 123: 3!, 112 & 122: 3!, 223 & 233: 3!, 331 & 311: 3! 111, 222, 333: 3 Candidates need not include this)
	(i)	**(c)**	**(i)(b)** – 3 If answer is not 24, this method must be explicitly stated in order to give M1A1ft = 24 ft their **(i)(b)**	M1 A1ft **[2]**	or 3! + 6×3 or 3! + 3!×3 or 6 + 3!×3!÷2! or 3! × 4 Complete correct method. Allow methods equiv to these. Only allow other methods if they appear correct	or 8 × 3 (Explanation: there are 8 possible orders starting with 1. Candidates need not include this)
	(ii)		eg 1123: $\frac{4!}{2!}$ × 3 alone allow M1 for $\frac{4!}{2!}$ × 3! alone eg 1122: $\frac{4!}{2!2!}$ × 3 alone allow M1 for $\frac{4!}{2!2!}$ × 3! alone Total = 54	M2 M2 A1 **[5]**	3! × $^4C_1$ × 3 or 3! × 12 M1 ÷ 2 M1dep (= 36) 3! × $^4C_2$ M1 ÷ 2 M1dep (= 18) Allow methods equiv to these, eg correctly listing cases Only allow other methods if they appear correct. NB 3×3×2×2 = 36 & 3×3×2 × 1 = 18 are incorrect methods unless clear justification given	This method only scores if 3×3×3×3 –…. is used: No. with 4 rep'ns = 3 M1 No. with 3 rep'ns = $\frac{4!}{3!}$ M1 × 6 (= 24) M1 or 8 × 3 M2 81–('3'+'24') or 81–27 M1 (allow 81–3 or 81–24) 18, 36 only score if a correct method seen,, or eg: 18 orders listed starting with "1" or 18 orders listed with two repetitions

(Q4, Jan 2013)